Name:

Smart Grids Technologies
Module 4, Lab 1 – 12/05/2025
Optimal Power Flow

1 Organization

1.1 Objectives

This lab session covers the basics of the optimal power flow problem. We assume you have completed the previous labs and are familiar with the different formulations for the load flow (the bus injection model (BIM) and branch flow model (BFM)). In this lab you will learn how to implement and solve an optimal power flow and apply the methods handled during the lectures to solve a deterministic scheduling problem. The lab aims to make you understand the implications of using different power flow models on the solution of the optimal power flow. First, we will formulate a simple problem, modelling the network as a copper plate. Then the SOCP relaxation is discussed and the assumptions under which it may be used are presented. Finally, the linearization using sensitivity coefficients is discussed. You are provided with a Jupyter notebook and a set of helper functions implemented in python. The notebook will help you to implement the different steps required to solve the questions in this document. You will be asked to formulate a set of constraints for the different methods discussed during the lectures, write an objective, perform some optimal power flow calculations, and interpret the obtained results. The optimal power flow problems will be formulated through CVXPY and solved using gurobi. Instructions for the installation are given below. Documentation on how to construct specific constraints or obtain advanced information (such as dual variable values) can be found here.

1.2 Evaluation

This report will not be graded; however, its submission is mandatory. The purpose of the questions within this document is to enhance your comprehension of the subject matter. Your acquired knowledge from the two laboratories of **Module 4** will be evaluated in a quiz scheduled for Monday, May 26th from 9:15 to 10:00. The deadline for submission of this report is Monday, May 19th at 23:55.

1.3 Installation of Gurobi

For the solution of the optimization problems, we recommend to use gurobi. The *requirements.txt* should install the *gurobipy* package, which will allow usage of gurobi through python. Additionally, a license is required. To generate a simple academic license, go to this link and request a license using your epfl account. For this you will need to create an account using your epfl email. Once you have, create a WLS Academic license. Finally, download the license and save it in a default folder. For Mac users, this should be *users/username*. Windows users can save the license in *C/users/username*.

2 Theory

This section of the lab introduces the optimal power flow problem and the different methods discussed during the lectures to solve it. If you are already familiar with the topics discussed here, feel free to skip this section, and proceed directly to Sec. 3.

2.1 The Optimal Power Flow Problem

In a general form, the optimal power flow problem (OPF) can be written as:

$$\min_{x} \quad C(x) \tag{1a}$$

$$\text{s.t.} \quad \text{resource constraints} \tag{1b}$$

$$\text{grid constraints} \tag{1c}$$

where x represents the set of optimization variables considered in the problem. In this laboratory, we will solve an optimal power flow problem, where the goal is to minimize the operation cost of a medium voltage (MV) network over a period divided in a set of time steps $t = 1, \ldots, T$. We will consider a system with battery energy storage systems (BESSs), uncontrollable loads and PV injections and gas turbines(GTs). We also allow to purchase electricity from the upper layer network (to which the MV system is connected in node 0, which we will consider to be the slack node). In this section, we recall and further detail how to model the interaction with the main grid, the gas turbines and the BESSs. The other injections are assumed to be known and fixed. Additionally, we will assume each node hosts at most 1 controllable resource (battery or gas turbine) and thus the controllable power injection supplied/absorbed at the node n is attributed to the controllable resource at that node. The net injection at a given node n is written as $S_n = P_n + jQ_n$, with $P_n = P_{cn} - P_{dn}$ and $Q_n = Q_{cn} - Q_{dn}$, where c refers to controllable injections and d refers to the uncontrollable injections.

BESS Model We assume an ideal lossless BESS. Additionally, we assume there is no cost associated with the utilisation of the battery. In this case, the model will consist of enforcing limits on the energy level and the power supplied or absorbed by the BESS. The energy level is constrained as following and includes an energy balance, a constraint on the final and initial state of charge and a constraint reflecting the limited energy capacity of the BESS.

$$SoC_n(t+1) = SoC_n(t) - P_n(t+1) \cdot \Delta t * \frac{1}{E_n^{cap}} \tag{2a}$$

$$SoC(1) = SoC^{init} - P_n(1) * \frac{1}{E_n^{cap}} \tag{2b}$$

$$SoC(T) \leqslant 1.1 SoC^{init} \tag{2c}$$

$$SoC(T) \geqslant 0.9 SoC^{init} \tag{2d}$$

$$SoC(t) \leqslant 0.9 \tag{2e}$$

$$SoC(t) \geqslant 0.1 \tag{2f}$$

$$C_n(t) = 0 \tag{2g}$$

The active and reactive power that can be supplied by a BESS is constrained by the converter connecting it to the network. Typically, the constraint can be represented by a circle $P^2 + Q^2 \leqslant S_{max}^2$ in the PQ plane. However, for simplicity, the constraint is often approximated through an inner square box approximation as shown in Figure 1.

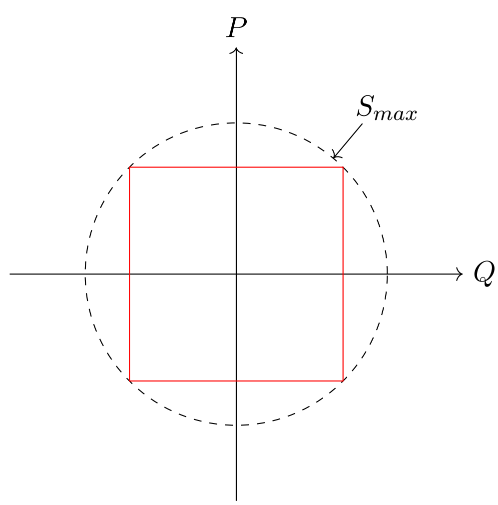

Figure 1: BESS power capability model.

Mathematically, the power capacity constraint can thus be modelled as:

$$P_n(t) \leqslant \frac{S_n^{max}}{\sqrt{2}} \tag{3a}$$

$$P_n(t) \geqslant -\frac{S_n^{max}}{\sqrt{2}} \tag{3b}$$

$$Q_n(t) \leqslant \frac{S_n^{max}}{\sqrt{2}} \tag{3c}$$

$$Q_n(t) \geqslant -\frac{S_n^{max}}{\sqrt{2}} \tag{3d}$$

Gas Turbines. Gas turbines convert gas to electricity. The conversion should be modelled to allow to account for their operational cost. Additionally, ramping constraints should be considered when modelling these resources. Here we will assume the gas turbines only provide active power and the reactive power injected is equal to zero. The gas turbines can thus be modelled using the following equations:

$$P_n(t) \leqslant P_n^{max} \tag{4a}$$

$$Q_n(t) = 0 \tag{4b}$$

$$P_n(t+1) - P_n(t) \leqslant \xi_n^{max} \Delta t \tag{4c}$$

$$P_n(t+1) - P_n(t) \geqslant \xi_n^{min} \Delta t \tag{4d}$$

$$C_n(t) = c^{gas} \frac{P_n(t)}{\eta^{GT}} \Delta t \tag{4e}$$

Power exchange with upper layer network. To model the allowed power exchange and account for the cost of buying and selling electricity, we introduce additional variables to differentiate between the import and export of electricity. This allows to model the difference in price for import and export. We will assume electricity can be sold at a third of the price for which it can be bought. The difference between the import and export gives the net injection at the slack node. This means that only one of the two variables $P_s^+(t)$ or $P_s^-(t)$ should be nonzero at any time t. This is represented in the following equations:

$$P_s^+(t) \geqslant 0 \tag{5a}$$
$$P_s^+(t) \leqslant P_{ex}^{max} \tag{5b}$$
$$P_s^-(t) \geqslant 0 \tag{5c}$$
$$P_s^-(t) \leqslant P_{ex}^{max} \tag{5d}$$
$$P_s(t) = P_s^+(t) - P_s^-(t) \tag{5e}$$
$$C_s(t) = c^{el}(t)(P_s^+(t) - \frac{1}{3}P_s^-(t))\Delta t \tag{5f}$$

Remaining nodes. For the nodes where no controllable generators are installed, the controllable power injections should be set to zero.

$$P_n(t) = 0 \tag{6a}$$
$$Q_n(t) = 0 \tag{6b}$$

2.2 The Second Order Cone Relaxation

In this lab, you will be asked to solve an optimal power flow using a second order cone relaxation. In this model, the equation determining the losses is relaxed to an inequality to make the problem convex. The equations forming this relaxation are listed here. For more information, you may refer to the slides. The relaxation you will implement is formulated starting from the branch flow model (BFM) discussed in Module 2. Figure 2 recalls this model and illustrates the variables used in the following equations. The equations constituting this model are given below for every generic line $L(i, j)$.

$$P_j - P_{dj} = \sum_k (P_{jk}) - P_{ij} + R_{ij} i_{Zij} + G_i v_i + G_j v_j \tag{7a}$$
$$Q_j - Q_{dj} = \sum_k (Q_{jk}) - Q_{ij} + X_{ij} i_{Zij} + B_i v_i + B_j v_j \tag{7b}$$
$$v_i = v_j + |\bar{Z}_{ij}|^2 i_{Zij} - 2\mathcal{R}(\underline{Z}_{ij}(\bar{S}_{ij} - \bar{Y}_{ij} v_i)) \tag{7c}$$
$$i_{Zij} \geqslant \frac{\bar{S}_{ij} - \bar{Y}_{ij} v_i}{v_i} \tag{7d}$$
$$0 \leqslant i_{Zij} \leqslant (I_{ij}^{max})^2 \tag{7e}$$
$$(V^{min})^2 \leqslant v_i \leqslant (V^{max})^2 \tag{7f}$$

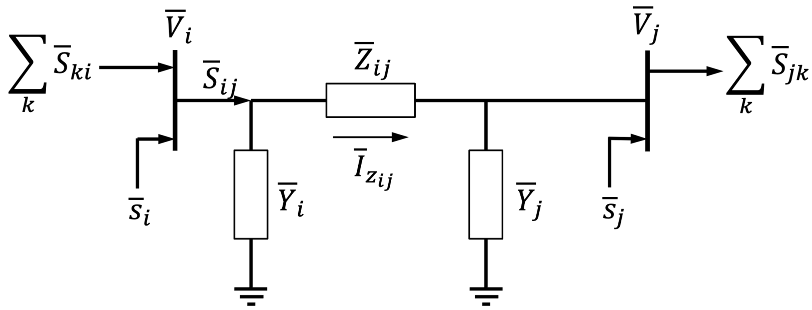

Figure 2: Branch Flow Model

Here we additionally define P_{dj} as the uncontrollable demand at node j. For an uncontrollable production, P_{dj} would be negative, thus reflecting the injection of power in the network.

2.3 Linearisation using Sensitivity Coefficients

The last method you will use to solve the optimal power flow problem, is a linearisation method using sensitivity coefficients to capture the variations of nodal voltages and line currents as a function of the nodal power injections. Assuming the system was linearized around a certain state $\bar{V}^*$ and sensitivity coefficients for the voltage and current magnitudes at every timestep were obtained as $\boldsymbol{K_{P,V}(t)}, \boldsymbol{K_{Q,V}(t)}, \boldsymbol{K_{P,I}(t)}, \boldsymbol{K_{P,I}(t)}$, we can write the constraints on the voltage and current magnitudes as:

$$V_{min} \leqslant |\bar{V}^*(t)| + \boldsymbol{K_{P,V}(t)} \cdot (P - \bar{P}) + \boldsymbol{K_{Q,V}(t)} \cdot (Q - \bar{Q}) \tag{8a}$$
$$V_{max} \geqslant |\bar{V}^*(t)| + \boldsymbol{K_{P,V}(t)} \cdot (P - \bar{P}) + \boldsymbol{K_{Q,V}(t)} \cdot (Q - \bar{Q}) \tag{8b}$$
$$-I_{max} \leqslant |\bar{I}^*(t)| + \boldsymbol{K_{P,V}(t)} \cdot (P - \bar{P}) + \boldsymbol{K_{Q,V}(t)} \cdot (Q - \bar{Q}) \tag{8c}$$
$$I_{max} \geqslant |\bar{I}^*(t)| + \boldsymbol{K_{P,I}(t)} \cdot (P - \bar{P}) + \boldsymbol{K_{Q,I}(t)} \cdot (Q - \bar{Q}) \tag{8d}$$

Additionally, we can compute sensitivity coefficients approximating the losses as a function of the nodal power injections. With these coefficients, the power at the slack can be computed as:

$$P_s(t) + \sum_n (P_n(t)) + L_p^*(t) = \boldsymbol{K_{P,P_s}(t)} \cdot (P - \bar{P}) + \boldsymbol{K_{Q,P_s}(t)} \cdot (Q - \bar{Q}) \tag{8e}$$

$$Q_s(t) + \sum_n (Q_n(t)) + L_q^*(t) = \boldsymbol{K_{P,Q_s}(t)} \cdot (P - \bar{P}) + \boldsymbol{K_{Q,Q_s}(t)} \cdot (Q - \bar{Q}) \tag{8f}$$

Note that although the current magnitudes should be positive, its sign can change, representing a change of direction of the current flow. Therefore, the constraint imposing a minimum on the current magnitude is bounded below by $-I_{max}$ instead of 0. The quantities P and Q refer to the vectors of concatenate injections $P_n = P_{cn} - P_{dn}$, $Q_n = Q_{cn} - Q_{dn}$ for all the nodes in the system (except for the slack node which is left out as the voltage is considered fixed).

3 Exercises

3.1 Introduction

In this laboratory, we consider a benchmark MV system. A set of profiles for uncontrollable injections are given, together with the parameters of the controllable resources present in the network. The network is visually represented in Figure 3. The network parameters are provided in a text file and helper functions are given to load the network data and construct the admittance matrix, retrieve the ampacities etc. Please note that the shunt admittances are given as positive values in per unit, but as they concern lines, their effect is capacitive and thus they should be modelled as negative admittances!

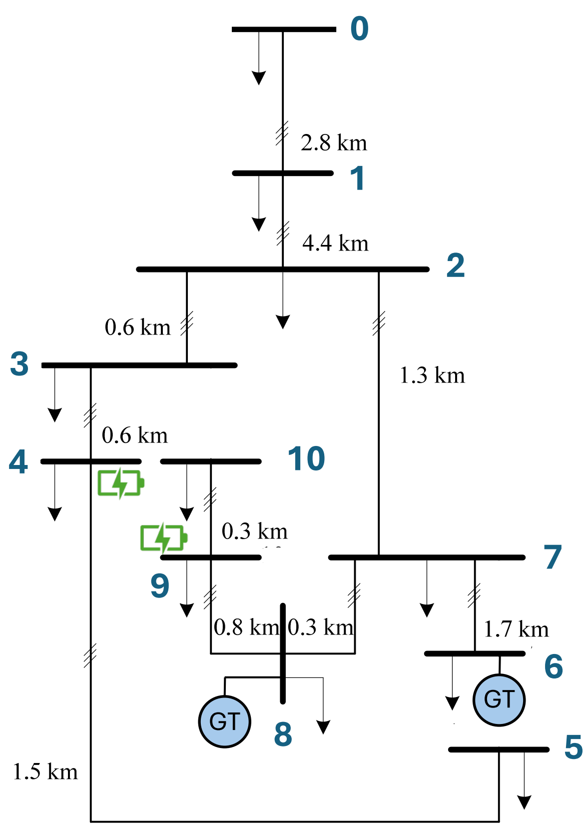

Figure 3: MV benchmark network.

3.2 Questions

You are now asked to answer the following questions. For questions requiring only coding, you do not need to copy the code in this report, but you may simply upload the code together with your report in the assignment. The first questions should be solved with only pen and paper. Consider the following basic OPF problem where we do not consider network constraints.

$$\min_{p_1,p_2} \quad 5p_1 + 15p_2 \tag{9a}$$
$$\text{s.t.} \quad p_1 + p_2 = 120 : \mu \tag{9b}$$
$$0 \leqslant p_1 : \lambda_1 \tag{9c}$$
$$0 \leqslant p_2 : \lambda_2 \tag{9d}$$
$$p_1 \leqslant 50 : \lambda_3 \tag{9e}$$
$$p_2 \leqslant 100 : \lambda_4 \tag{9f}$$

Q1/ Write the Lagrangian for this OPF problem. Remember to first reformulate the optimization problem in standard form.

[A1]

The Lagrangian is given by:

$$L(p_1, p_2, \mu, \lambda_i) = 5p_1 + 15p_2 - \lambda_1 p_1 - \lambda_2 p_2 + \lambda_3(p_1 - 50) + \lambda_4(p_2 - 100) + \mu(p_1 + p_2 - 120) \tag{10}$$

Q2/ Formulate the corresponding KKT conditions

[A2]

The KKT conditions are the following:

$$\frac{dL}{dp_1} = 5 - \lambda_1 + \lambda_3 + \mu = 0 \tag{11a}$$

$$\frac{dL}{dp_2} = 15 - \lambda_2 + \lambda_4 + \mu = 0 \tag{11b}$$

$$0 \leqslant p_1 \leqslant 50 \tag{11c}$$

$$0 \leqslant p_2 \leqslant 100 \tag{11d}$$

$$p_1 + p_2 = 120 \tag{11e}$$

$$\lambda_1(-p_1) = 0 \tag{11f}$$

$$\lambda_2(-p_2) = 0 \tag{11g}$$

$$\lambda_3(p_1 - 50) = 0 \tag{11h}$$

$$\lambda_4(p_2 - 100) = 0 \tag{11i}$$

Q3/ Solve the optimization problem and use the KKT conditions to obtain the optimal values for primal and dual variables as well as the optimal objective. What is the significance of the different dual variables?

[A3]

The system of equations can be solved through the following steps. As the first generator is cheaper, we can trivially find that it will be fully used. Therefore, we get $p_1 = 50$. From the power balance then follows that $p_2 = 70$. Through complimentary slackness, we then find that $\lambda_1, \lambda_2 and \lambda_4$ must all be 0. Finally, combining the two stationarity conditions we get $\lambda_3 = 10$ and filling thiss in one of the two gives us $\mu = -15$. Finally, we can interpret the dual variables. Non-zero dual variables are associated to binding constraints and show us how the objective of the minimization problem would decrease if the right hand side of the associated equation would change (written in standard form). Therefore, we can interpret λ_3 as the value of increasing the capacity of generator 1 (note that it is equal to the difference in cost between the two generators), while μ represent the cost of supplying an additional unit of load. Given that μ is negative, we know that increasing the load will lead to an increase in the total cost.

The controllable resources consist of BESSs for which we need to model the state of energy and power capacity and gas turbines subject to ramping constraints. Additionally, to account for the cost of importing of exporting electricity, we differentiate between positive and negative power exchange at the slack node.

Q4/ Create and list the variables required to model the controllable resources in the system. As an example, the variables describing the reactive nodal power injections are given.

[A4]

The variables are defined in the solution notebook.

In Section 2.1 we recall the models for the controllable resources considered in this exercise.

Q5/ Fill in the function constructing the constraints related to the operation of the different controllable resources.

[A5]

The correct constraints are given in the code. Pay attention that for the ramping a value was given relative to the nominal capacity of the gas turbines and for the power change over one hour. To recover the ramping limits ξ^{max} and ξ^{min} you need to compute the maximal power change when increasing or decreasing the power output. The limits the become $\xi^{max} = ramping * P^{cap}$ and $\xi^{min} = -ramping * P^{cap}$.

We now need to define the objective for the optimal power flow problem we want to solve. As mentioned earlier, the goal is to minimize the operation cost of the system over a day, divided in T timesteps. Comments are provided to guide you through the different cost contributions making up the objective function.

Q6/ Write the function creating the minimization objective for the OPF problem. A term penalising losses and high power injections is already included. Why is it important that the cost increases as a function of the losses when using the SOCP relaxation?

[A6]

The objective is given and translates the cost functions for the electricity import/export and the operation of the gas turbines in python code. The objective needs to be increasing as a function of the losses due to the relaxation of the constraint on the losses. By increasing the cost function as a function of the losses, the optimal solution will be one that minimises the losses, thus pushing the inequality constraint towards equality. Additionally, note that here we do not explicitly add the losses in the objective function, but add the norm of the power injections and have cost functions that will increase when more power is injected in the network (and thus indirectly increasing with increasing losses).

You are now asked to solve a simple optimal power flow accounting for the resource constraints and the objective you just defined without including grid constraints. However, to ensure the power balance, we model the power flow as a copper plate.

Q7/ Write the constraint that ensures the network is modelled as a copper plate. (This means we are only interested in active power).

[A7]

For the copper plate approximation, the only constraint required on top of the resource constraints is an active power balance. Reactive power is not considered in this case and can be seen as a free variable of the problem.

The next question concerns the SOCP relaxation for the optimal power flow. A number of constraints are already given. You are asked to complete the function writing the constraints. For the constraint relaxing the losses, pay attention to the way you formulate it. It needs to be transformed to a cone using the approach presented above to ensure the solver realises the constraint is convex. To this end, you will need to transform the quadratic constraint using the following trick:

$$\|x\|_2^2 \leqslant s \cdot t \iff \left\| \begin{matrix} 2x \\ s - t \end{matrix} \right\|_2 \leqslant s + t \tag{12}$$

Additionally, we recommend referring the power balance constraints around the node *downstream* of the line (i.e. the node j in the figure shown above).

Helper matrices have been provided to help you link the lines with the corresponding nodes to associate the line flows to the correct nodal voltages. The reactive power balance is given as an example.

Q8/ Complete the function defining the constraints for the SOCP relaxation model.

[A8]

The constraints for the SOCP model are implemented in the notebook. These translate the model presented in the slides and lab notes.

Now we solve the OPF problem using the SOCP model. Plot the power injections, nodal voltages and state of charge evolution.

Q9/ Can you explain the profiles of the different controllable injections? Why is it different from the copper plate approximation used above? Are there binding constraints? If yes, can you determine which one? Hint: In cvxpy, you can request the values of the dual variables associated to given constraints. As you passed the constraints as a list, you need to think about which constraint should be binding and at which timestep this would occur. This will allow you to determine the index of the constraint you expect to be binding and to check the duals for that specific constraint. For example, to get the value of the dual variables associated to the first constraint: $dual\_value = constraints[0].dual\_value$

[A9]

We can make three main observations when considering the profiles of the power injections. The gas turbines are operated mostly during the peak hours and ramp up/down for the off-peak times. This is due to the price difference of electricity and gas (gas has a constant price during the day, while the electricity price is high at peak hours. Additionally, we can observe that the battery power contribution is quite limited. Indeed it has a comparable power capacity, but the energy capacity restricts the battery from playing a important role. We see nevertheless the battery is charged during off-peak hours to provide energy during peak hours. Finally, comparing the SOCP solution with the copper plate approximation, we can observe that the power output of the gas turbine at node 6 has been reduced by approximately 0.5MW. This indicates that there might be a binding constraint. The first suspect should be the line connecting the node with this gas turbine to the rest of the network (this is line 5 (zero-indexed). By finding the constraint associated to this lines current limit at the times where the power output is reduced and verifying the value of the dual variable, we can see that the dual value is non zero, thus indicating that this constraint is active and thus binding.

To verify the accuracy of the result of an optimal power flow computation, we can compute the slack power using the non-approximated power flow equations through a load flow. To this end, we solve the load flow equations while imposing the optimal controllable injections obtained from the solution.

Q10/ Check the accuracy of the solution. Is this expected?

[A10]

To verify the accuracy of the obtained solution, we solve the load flow equations imposing the optimal power injections. We can then compare the slack power obtained to the slack power predicted by our optimisation problem. We can see that the error is very low. This is expected as the SOCP approximation should be tight due to the objective, which is increasing with increasing losses.

Next, we will solve the optimal power flow problem using an approximation based on a linearization of the grid model through sensitivity coefficients. The power balance at the slack node is already given. The sensitivity coefficients used to compute the losses can be obtained through a linear combination of the current and voltage sensitivities. Check their computation in the *BasicFunctionsOPF.py* file.

Q11/ Complete the function constructing the electricity grid constraints using the linearization through sensitivity coefficients.

[A11]

The constraints are given in the notebook.

Q12/ Solve the problem using the linearised electricity network model. Study the results. Do they correspond to the ones obtained using the SOCP model? Why (not) ?

[A12]

The solution does not correspond. This can be understood by thinking about the problem that is solved by the linearized OPF. This is a linear problem that is a local approximation of the non-approximated OPF problem. It is only accurate in the neighborhood of the linearization point. The first steps will typically be quite large and not yield accurate solutions.

Finally, solve the optimal power flow problem in a sequential way by iteratively obtaining a linearized model around the system state and solving the resulting linear program.

Q13/ Fill in the gaps in the code and analyse the solution. How does this compare with the solution from the SOCP?

[A13]

By iteratively linearizing the problem, we can get a more accurate approximation of the original problem. However, as we still have a linear approximation, convergence issues can occur by selecting solutions far from the initial point that are feasible for the linearized problem, but not for the original problem. The new linearization point (computed by the load flow) will then yield a very different linearized problem and the differences between subsequent iterations remain large. To mitigate this behaviour, we can limit the step size in each iteration (by limiting the change in the power injections). This ensure the approximation stays closer to the real problem and we converge faster. Due to the iterative linearization, we end up in a local minimum and do not find the same solution as with the SOCP relaxation.